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$O(5)\times U(1)$ Electroweak Gauge Theory and the Relevance of the Cabibbo Angle in *CP* Violation in *K* Decays

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Some of the relevant mathematics of $O(5)\times U(1)$ electroweak gauge theory is briefly sketched. The $O(5)\times U(1)$ model is presented. To facilitate the discussion of *CP* violation in *K* decays, the relevant Lagrangian is given in several alternative forms. It is shown that in the *CP*-violating part of the Lagrangian, by a redefinition of quark phases, the coupling of the *CP* eigenstates K_1 and K_2 cannot be broken. However, if the Cabibbo angle were not present, the states K_1 and K_2 would decouple and the theory would become *CP*-invariant. Such a result was also reported by Deshpande *et al.*, working with a different formalism. Relating the mixing parameters θ and ϕ to the parameters ε_1 and ε_2, it is shown that when $\varepsilon_1=\varepsilon_2=\varepsilon$, ε reduces to the usual *CP*-violating and *CPT*-conserving parameter.

1. INTRODUCTION

The K^0 and $\bar{K}^0$ mesons are charge conjugates of one another and possess strangeness +1 and −1, respectively. However, they do not have definite lifetimes for weak decay, nor do they have any definite masses. Since the weak interactions do not conserve strangeness, there exist two linear combinations of the states $|K^0\rangle$ and $|\bar{K}^0\rangle$, namely $|K_S^0\rangle$ and $|K_L^0\rangle$, which have definite masses and lifetimes. The short-lived K_S^0 meson decays into two predominant modes $\pi^+\pi^-$ and $\pi^0\pi^0$, each with the *CP* eigenvalue +1, whereas the long-lived K_L^0 meson has among its decay modes $\pi^+\pi^-\pi^0$ and $\pi^0\pi^0\pi^0$, which are eigenstates of *CP* with the eigenvalue −1. Since with the conventional choice of phase we can write

$$CP|K^0\rangle=-|\bar{K}^0\rangle,\qquad CP|\bar{K}^0\rangle=-|K^0\rangle \tag{1}$$

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we have

$$|K_1^0\rangle = \frac{1}{\sqrt{2}}(|K^0\rangle + |\bar{K}^0\rangle); \qquad CP|K_1^0\rangle = -|K_1^0\rangle$$
$$|K_2^0\rangle = \frac{1}{\sqrt{2}}(|K^0\rangle - |\bar{K}^0\rangle); \qquad CP|K_2^0\rangle = +|K_2^0\rangle \tag{2}$$

where $|K_1^0\rangle$ and $|K_2^0\rangle$ are defined in (2). However, Christenson *et al.* (1964) observed that there is a small but finite probability for the decay $K_L^0 \to \pi^+\pi^-$, in which the final state has the *CP* eigenvalue +1. Thus, we cannot identify K_L^0 with K_1^0 and K_S^0 with K_2^0. In fact, different mixings of K_1^0 and K_2^0 correspond to K_L^0 and K_S^0; see equations (40) and (41).

Since its discovery, several attempts have been made to explain the *CP* violation in *K* decays (e.g., Chau, 1983; Frère, 1985; Grimus, 1987; Donoghue *et al.*, 1987; Ecker, 1987). However, none of the theories put forward so far is regarded as an acceptable one and thus *CP* violation still remains a mystery.

Another outstanding mystery of weak interaction physics is the existence of the Cabibbo angle (Cabibbo, 1963), tieing together many otherwise fragmentary empirical peculiarities of the theory, a good description of which is given by Commins (1973). The Cabibbo angle is an essential ingredient of the weak interaction theory and is an empirical parameter introduced by Cabibbo still awaiting explanation in terms of more fundamental quantities.

It is the purpose of this paper to trace a possible relation between the existence of the Cabibbo angle and the *CP* noninvariance in *K* decays. A proper framework for such an investigation is a unified gauge model. In this work we employ the $O(5)\times U(1)$ electroweak gauge theory and find that the existence of the Cabibbo angle forms a necessary condition for the *CP* violation in *K* decays.

Regarding the $O(5)\times U(1)$ electroweak gauge theory, it suffices to say that the group $O(5)$ is anomaly-free and economical in the number of gauge bosons that we associate with each of its generators. We have assigned the left-handed quarks $Q_L^T = (u, d_c, s_c, c)_L$ to the four-dimensional spinorial representation of the group $O(5)$, whereas the right-handed particles are taken to be the singlets of the group where d_c and s_c are the Cabibbo rotated quarks defined in equation (15), The theory has three sets of gauge bosons: (1) analogs of the Glashow (1961), Weinberg (1967), and Salam (1968) (GWS) model, and additional (2) charged and (3) neutral gauge bosons as compared to the GWS model.

Section 2 briefly sketches the relevant mathematics. Section 3 discusses the $O(5)\times U(1)$ model. Section 4 establishes our main thesis and Section 5 discusses the results.

2. THE RELEVANT MATHEMATICS

To set our notation and indicate the particular representation used in this work, we briefly sketch some of the relevant mathematics. The method of constructing the spinorial representations in higher dimensions for rotation groups is discussed by Brauer and Weyl (1935). Following their method, we construct the four-dimensional spinorial representations of the five-dimensional rotation group $O(5)$. We take a set of five 4×4 Hermitian anticommuting matrices Γ_a:

$$\Gamma_a^\dagger = \Gamma_a, \qquad \{\Gamma_a, \Gamma_b\} = 2\delta_{ab}, \qquad a, b = 1, \ldots, 5 \tag{3}$$

and

$$\begin{aligned} &\Gamma_1 = \sigma_1^{(1)} \times \sigma_1^{(2)}, \qquad \Gamma_2 = \sigma_1^{(1)} \times \sigma_2^{(2)}, \qquad \Gamma_3 = \sigma_3^{(1)} \times 1 \\ &\Gamma_4 = \sigma_1^{(1)} \times \sigma_3^{(2)}, \qquad \Gamma_5 = \sigma_2^{(1)} \times 1 \end{aligned} \tag{4}$$

The superscripts (1) and (2) refer to two distinct sets of Pauli matrices; the symbol $\times$ stands for their direct product; and 1 stands for the 2×2 unit matrix. The generators are given by

$$F_{ab} = -\tfrac{1}{2} i \Gamma_a \Gamma_b, \qquad a \neq b \tag{5}$$

The restriction is imposed due to the antisymmetry of F_{ab}. Explicitly the matrices read

$$\begin{aligned} &\Gamma_1 = \begin{pmatrix} 0 & \sigma_1 \\ \sigma_1 & 0 \end{pmatrix}, \qquad \Gamma_2 = \begin{pmatrix} 0 & \sigma_2 \\ \sigma_2 & 0 \end{pmatrix}, \qquad \Gamma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \\ &\Gamma_4 = \begin{pmatrix} 0 & \sigma_3 \\ \sigma_3 & 0 \end{pmatrix}, \qquad \Gamma_5 = \begin{pmatrix} 0 & -i\times 1 \\ i\times 1 & 0 \end{pmatrix} \end{aligned} \tag{6}$$

The generators are given as follows:

$$\begin{aligned} &F_{12} = \tfrac{1}{2}\begin{pmatrix} \sigma_3 & 0 \\ 0 & \sigma_3 \end{pmatrix}, \qquad F_{13} = \tfrac{1}{2}\begin{pmatrix} 0 & i\sigma_1 \\ -i\sigma_1 & 0 \end{pmatrix}, \qquad F_{14} = \tfrac{1}{2}\begin{pmatrix} -\sigma_2 & 0 \\ 0 & -\sigma_2 \end{pmatrix} \\ &F_{15} = \tfrac{1}{2}\begin{pmatrix} \sigma_1 & 0 \\ 0 & -\sigma_1 \end{pmatrix}, \qquad F_{23} = \tfrac{1}{2}\begin{pmatrix} 0 & i\sigma_2 \\ -i\sigma_2 & 0 \end{pmatrix}, \qquad F_{24} = \tfrac{1}{2}\begin{pmatrix} \sigma_1 & 0 \\ 0 & \sigma_1 \end{pmatrix} \\ &F_{25} = \tfrac{1}{2}\begin{pmatrix} \sigma_2 & 0 \\ 0 & -\sigma_2 \end{pmatrix}, \qquad F_{34} = \tfrac{1}{2}\begin{pmatrix} 0 & -i\sigma_3 \\ i\sigma_3 & 0 \end{pmatrix}, \qquad F_{35} = \tfrac{1}{2}\begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix} \\ &F_{45} = \tfrac{1}{2}\begin{pmatrix} \sigma_3 & 0 \\ 0 & -\sigma_3 \end{pmatrix} \end{aligned} \tag{7}$$

The generators satisfy the following commutation relation:

$$[F_{ab}, F_{cd}] = i(\delta_{ac}F_{bd} - \delta_{bc}F_{ad} + \delta_{bd}F_{ac} - \delta_{ad}F_{bc}) \tag{8}$$

forming the corresponding Lie algebra.

For our purposes it is convenient to consider the algebra in a different basis

$$\{F_i, F_{45}, F_i^{\pm}\}, \qquad i=1,2,3 \tag{9}$$

defined as

$$F_1=F_{23}, \qquad F_2=F_{13}, \qquad F_3=F_{12} \tag{10}$$

$$F_1^{\pm}=F_{14}\pm iF_{15}, \qquad F_2^{\pm}=F_{24}\pm iF_{25}, \qquad F_3^{\pm}=F_{34}\pm iF_{35} \tag{11}$$

among the above set of generators.

Using equation (8), in particular, the following commutation relations can be established:

$$\begin{aligned}&[F_i^{\pm}, F_i^{\mp}]=\pm 2F_{45} \quad (i \text{ not summed})\\ &[F_{45}, F_i^{\pm}]=\pm F_i^{\pm}\end{aligned} \tag{12}$$

$$[F_{45}, F_i]=0 \tag{13}$$

$$[F_i, F_j]=i\in_{ijk}F_k \tag{14}$$

From equations (12) and (14) we see that for every value of $i(=1,2,3)$ the set of generators $\{F_{45}, F_i^{\pm}\}$ and $\{F_i\}$ form $Su(2)$ subalgebras. Since the charge operator in equation (16) is defined using the generator F_{45}, equations (12) and (13) indicate that the generators F_i and $F_i^{\pm}$ are the eigenvectors of the charge operator with the eigenvalues 0 and ± 1; the charge is invariant under the group $O(5)$ and eventually under the larger group $O(5)$ and $U(1)$.

3. THE $O(5)\times U(1)$ MODEL

As we are seeking a relation between the Cabibbo angle and the CP violation in K decays, it is sufficient to use the simplest relation in which the Cabibbo angle enters. From the Kobayashi and Maskawa (1973) matrix, it is easily seen that such a relation involves only four flavors, namely u, d, s, c. Accordingly, in the present work we develop the theory in terms of only these flavors and employ the four-dimensional spinorial representation of the group $O(5)$, to which the left-handed quark multiplet $Q_L^T=(u, d_c, s_c, c)_L$ is assigned, whereas the right-handed particles u_R, d_R, s_R, c_R are taken to be the singlets of the group, where d_c and s_c are the Cabibbo rotated quarks and are given by the relations (θ_c is the Cabibbo angle)

$$d_c=\cos\theta_c\, d+\sin\theta_c\, s, \qquad S_c=-\sin\theta_c\, d+\cos\theta_c\, s \tag{15}$$

In this model we have ten gauge fields $W_{ij}(i<j=1,\ldots,5)$ and a singlet vector gauge field transforming as the $O(5)$ and $U(1)$ generators, respectively. We give in Table I the eigenvalues of the operators F_{45} and F_0 along with their charges for the flavors u, d, s, c.

Table I. The Relevant Quantum Numbers of u, d, s, and c

	Quarks			
	$u_{L,R}$	$d_{L,R}$	$s_{L,R}$	$c_{L,R}$
Q	$\frac{2}{3}$	$-\frac{1}{3}$	$-\frac{1}{3}$	$\frac{2}{3}$
Y_{45}	$\frac{1}{2}, 0$	$-\frac{1}{2}, 0$	$-\frac{1}{2}, 0$	$\frac{1}{2}, 0$
Y_0	$\frac{1}{3}, \frac{4}{3}$	$\frac{1}{3}, -\frac{2}{3}$	$\frac{1}{3}, -\frac{2}{3}$	$\frac{1}{3}, \frac{4}{3}$

The eigenvalue Y_{45} of the operator F_{45} is taken to be zero for the right-handed particles u_R, d_R, s_R, and c_R, since they are the singlets and do not belong to the four-dimensional representation of the group $O(5)$. In terms of the $O(5)\times U(1)$ generators the charge operator is given as

$$Q = F_{45} + \tfrac{1}{2}F_0 \tag{16}$$

Because of (10) and (11), it is possible to define a basis for the gauge bosons such that in the Lagrangian (22) certain combinations of the gauge fields, for instance, $(1/\sqrt{2})(W_\mu^{24} + iW_\mu^{25})$, can be universally coupled to the charged currents $\bar{u}\gamma^\mu \cdot \frac{1}{2}(1+\gamma^5)d$ and $\bar{c}\gamma^\mu \cdot \frac{1}{2}(1+\gamma^5)s$ rather than the separate ones W_μ^{24} and W_μ^{25}. We define

$$\begin{gathered} F_C = F_{12}, \qquad F_D = F_{13}, \qquad F_E = F_{23}, \qquad F_F = F_{45} \\ F_U^\pm = \frac{1}{\sqrt{2}} F_1^\pm, \qquad F_V^\pm = \frac{1}{\sqrt{2}} F_3^\pm, \qquad F_W^\pm = \frac{1}{\sqrt{2}} F_2^\pm \end{gathered} \tag{17}$$

The corresponding basis for the gauge fields is taken as

$$\begin{gathered} C_\mu = W_\mu^{12}, \qquad D_\mu = W_\mu^{13}, \qquad E_\mu = W_\mu^{23}, \qquad F_\mu = W_\mu^{45} \\ U_\mu^\pm = \pm\frac{i}{\sqrt{2}}(W_\mu^{14} \mp iW_\mu^{15}), \qquad V_\mu^\pm = \mp\frac{i}{\sqrt{2}}(W_\mu^{34} \mp iW_\mu^{35}) \\ W_\mu^\pm = \frac{1}{\sqrt{2}}(W_\mu^{24} \mp iW_\mu^{25}) \end{gathered} \tag{18}$$

Denoting the gauge couplings for the groups $O(5)$ by g and for $U(1)$ by $\frac{1}{2}g'$, we express the couplings of the fermion currents (ψ representing the quark fields) to the gauge bosons [with $\bar{a}$ defined as the Dirac conjugate of a and the abbreviations $a_L = \frac{1}{2}(1+\gamma^5)a$, $a_R = \frac{1}{2}(1-\gamma^5)a$, and $\bar{a}\gamma^\mu b \to \bar{a}\gamma^\mu \cdot \frac{1}{2}(1+\gamma^5)b$] by the following interaction Lagrangian:

$$\begin{aligned} L_{\text{int}} = g \sum_{i<j} & (\bar{\psi}_L \gamma^\mu F_{ij} W_\mu \psi_L) \\ & + \tfrac{1}{2}g'[\tfrac{1}{3}(\bar{u}_L\gamma^\mu W_\mu^0 u_L + \bar{d}_L\gamma^\mu W_\mu^0 d_L \\ & + \bar{S}_l\gamma^\mu W_\mu^0 S_l + \bar{C}_l\gamma^\mu W_\mu^0 C_l) \\ & + (\bar{U}_r\gamma^\mu \tfrac{4}{3} W_\mu^0 U_r - \bar{d}_R\gamma^\mu \tfrac{2}{3} d_R - \bar{S}_r\gamma^\mu \tfrac{2}{3} W_\gamma^0 S_r + \bar{C}_l\gamma^\mu \tfrac{4}{3} W_\mu^0 C_l)] \end{aligned} \tag{19}$$

Furthermore, defining

$$J_\mu(em) = (\tfrac{2}{3}\bar{u}\gamma^\mu u - \tfrac{1}{3}\bar{d}\gamma^\mu d - \tfrac{1}{3}\bar{s}\gamma^\mu s + \tfrac{2}{3}\bar{c}\gamma^\mu c) \tag{20}$$

$$J_{\mu,L} = (\bar{u}_L\gamma^\mu u_L - \bar{d}_L\gamma^\mu d_L - \bar{s}_L\gamma^\mu s_L + \bar{c}_L\gamma^\mu c_L) \tag{21}$$

we can write the interaction Lagrangian as

$$\begin{aligned} L_{\text{int}} = {} & g' W_\mu^0 J_\mu(em) + \tfrac{1}{2}(gF_\mu - g' W_\mu^0)J_{\mu,L} \\ & + \tfrac{1}{2}gC_\mu(\bar{u}_L\gamma^\mu u_L - \bar{d}_{cL}\gamma^\mu d_{cL} + \bar{s}_{cL}\gamma^\mu s_{cL} - \bar{c}_L\gamma^\mu c_L) \\ & + \tfrac{1}{2}giD_\mu(\bar{u}_L\gamma^\mu c_L + \bar{d}_{cL}\gamma^\mu s_{cL} - \bar{s}_{cL}\gamma^\mu d_{cL} - \bar{c}_L\gamma^\mu u_L) \\ & + \tfrac{1}{2}gE_\mu(\bar{u}_L\gamma^\mu c_L - \bar{d}_{cL}\gamma^\mu s_{cL} - \bar{s}_{cL}\gamma^\mu d_{cL} + \bar{c}_L\gamma^\mu u_L) \\ & + \frac{1}{\sqrt{2}}gU_\mu^+(\bar{u}_L\gamma^\mu s_{cL} - \bar{c}_L\gamma^\mu d_{cL}) + \text{H.c.} \\ & + \frac{1}{\sqrt{2}}gW_\mu^+(\bar{u}_L\gamma^\mu d_{cL} + \bar{c}_L\gamma^\mu s_{cL}) + \text{H.c.} \end{aligned} \tag{22}$$

Inserting the relations (15) for the Cabibbo rotated quarks and defining $\widetilde{E_\mu}$ and $\widetilde{C_\mu}$ by the equations

$$\begin{aligned} \widetilde{E_\mu} &= \cos 2\theta_C\, E_\mu + \sin 2\theta_C\, C_\mu \\ \widetilde{C_\mu} &= -\sin 2\theta_C\, E_\mu + \cos 2\theta_C\, C_\mu \end{aligned} \tag{23}$$

we can recast (22) as

$$\begin{aligned} L_{\text{int}} = {} & g' W_\mu^0 J_\mu(em) + \tfrac{1}{2}(gF_\mu - g' W_\mu^0)J_{\mu,L} \\ & + \tfrac{1}{2}gC_\mu(\bar{u}_L\gamma^\mu u_L - \bar{c}_L\gamma^\mu c_L) \\ & + \tfrac{1}{2}giD_\mu(\bar{u}_L\gamma^\mu c_L - \bar{c}_L\gamma^\mu u_L) \\ & + \tfrac{1}{2}gE_\mu(\bar{u}_L\gamma^\mu c_L + \bar{c}_L\gamma^\mu u_L) \\ & + \tfrac{1}{2}g\widetilde{C_\mu}(\bar{s}_L\gamma^\mu s_L - \bar{d}_L\gamma^\mu d) \\ & - \tfrac{1}{2}g\widetilde{E_\mu}(\bar{d}_L\gamma^\mu s_L + \bar{s}_L\gamma^\mu d_L) + \tfrac{1}{2}giD_\mu(\bar{d}_L\gamma^\mu s_L - \bar{s}_L\gamma^\mu d_L) \\ & + \frac{1}{\sqrt{2}}gU_\mu^+(\bar{u}_L\gamma^\mu s_{cL} - \bar{c}_L\gamma^\mu d_{cL}) + \text{H.c.} \\ & + \frac{1}{\sqrt{2}}gV_\mu^+(\bar{u}_L\gamma^\mu s_{cL} + \bar{c}_L\gamma^\mu d_{cL}) + \text{H.c.} \\ & + \frac{1}{\sqrt{2}}gW_\mu^+(\bar{u}_L\gamma^\mu d_{cL} + \bar{c}_L\gamma^\mu s_{cL}) + \text{H.c.} \end{aligned} \tag{24}$$

From equation (23) we see that $\widetilde{E}_\mu$ is defined as a linear combination of $E_\mu(\equiv W^{23})$, $C_\mu(\equiv W^{12})$, and $D_\mu(\equiv W^{13})$, which transform as the $O(5)$ generators F_{23}, F_{12}, and F_{13}, respectively. Also, from equation (14) we know that these are the generators of the $SU(2)$ group. Hence, introducing angles θ and ϕ and defining D'_μ and E'_μ as the $SU(2)$ transformed states by the relations

$$\begin{aligned} E_\mu &= \cos(\theta/2)E'_\mu - e^{-i\phi}\sin(\theta/2)D'_\mu \\ D_\mu &= e^{i\phi}\sin(\theta/2)E'_\mu + \cos(\theta/2)D'_\mu \end{aligned} \tag{25}$$

and inserting them for E_μ and D_μ in equation (24), it reduces to

$$\begin{aligned} L_{\text{int}} = {} & g'W^0_\mu J_\mu(em) + \tfrac{1}{2}(gF_\mu - g'W^0_\mu)J_{\mu,L} \\ & + \tfrac{1}{2}gC_\mu(\bar{u}_L\gamma^\mu u_L - \bar{c}_L\gamma^\mu c_L) + \tfrac{1}{2}giD_\mu(\bar{u}_L\gamma^\mu c_L - \bar{c}_L\gamma^\mu u_L) \\ & + \tfrac{1}{2}gE_\mu(\bar{u}_L\gamma^\mu c_L + \bar{c}_L\gamma^\mu u_L) + \tfrac{1}{2}g\widetilde{C}_\mu(\bar{s}_L\gamma^\mu s_L - \bar{d}_L\gamma^\mu d_L) \\ & - \tfrac{1}{2}gE'_\mu[\cos(\theta/2)(\bar{d}_L\gamma^\mu s_L + \bar{s}_L\gamma^\mu d_L) \\ & + e^{i\phi}\sin(\theta/2)i(s_L\gamma^\mu d_L - \bar{d}_L\gamma^\mu s_L) \\ & - \tfrac{1}{2}gD'_\mu[-e^{-i\phi}\sin(\theta/2)(\bar{d}_L\gamma^\mu s_L + \bar{s}_L\gamma^\mu d_L) \\ & + \cos(\theta/2)i(\bar{s}_L\gamma^\mu d_L - \bar{d}_L\gamma^\mu s_L) \\ & + \frac{1}{\sqrt{2}}gU^+_\mu(\bar{u}_L\gamma^\mu s_{cL} - \bar{c}_L\gamma^\mu d_{cL}) + \text{H.c.} \\ & + \frac{1}{\sqrt{2}}gV^+_\mu(\bar{u}_L\gamma^\mu s_{cL} + \bar{c}_L\gamma^\mu d_{cL}) + \text{H.c.} \\ & + \frac{1}{\sqrt{2}}gW^+_\mu(\bar{u}_L\gamma^\mu d_{cL} + \bar{c}_L\gamma^\mu s_{cL}) + \text{H.c.} \end{aligned} \tag{26}$$

It may be remarked that equations (22) and (23) are equivalent expressions and (26) is obtained by the introduction of additional parameters θ and ϕ. These equations are given here for our later discussion of CP violation in Section 4. To establish a relation between the masses of the additional gauge bosons, we work out their mass Lagrangian, which in turn requires the spontaneous symmetry breaking of the Lagrangian (22) up to the $SU(2)\times U(1)$ level.

To achieve this objective, following Li (1974), we break the symmetry down to $SU(2)\times U(1)$ in two steps. We introduce two Higgs scalar multiplets η and ξ transforming as vectors, for instance, $\eta_i \to \eta_i + \varepsilon_{ij}\eta_j$, under the group $O(5)$ and choose the following expectation values for η and ξ:

$$\langle\eta_i\rangle = V_1\delta_{i1}, \qquad \langle\xi_i\rangle = \delta_{3i}V_3 \tag{27}$$

which break the symmetry spontaneously to the required level and eventually minimize the relevant potential. The Higgs coupling to the vector boson fields, for instance, is given by

$$\mathscr{L}_{W\eta} = \tfrac{1}{2}(\partial\mu\,\eta_i - gW_{\mu ik}\eta_k) \times (\partial^\mu \eta_i - gW_{\mu il}\eta_l) \tag{28}$$

Substituting the relation (21) into (28) yields the boson mass term as

$$\tfrac{1}{2}g^2(W_{\mu ik}\langle\eta_k\rangle W_{\mu il}\langle\eta_l\rangle) \tag{29}$$

Using definitions (18), we find for the additional gauge boson mass term

$$\tfrac{1}{2}g^2V_1^2(C_\mu^2 + D_\mu^2 + U_\mu^+U_\mu^- + U_\mu^-U_\mu^+) \tag{30}$$

and a similar substitution of (27) for ξ in (30) reduces to

$$\tfrac{1}{2}g^2V_3^2(D_\mu^2 + E_\mu^2 + V_\mu^+V_\mu^- + V_\mu^-V_\mu^+) \tag{31}$$

Thus, the mass Lagrangian for the additional gauge bosons can be written as follows:

$$\begin{aligned}\mathscr{L}_M&(\text{additional gauge bosons})\\ &= \tfrac{1}{2}g^2(V_1^2 + V_3^2)D_\mu^2 + \tfrac{1}{2}g^2V_1^2C_\mu^2 + \tfrac{1}{2}g^2V_3^2E_\mu^2\\ &\quad + \tfrac{1}{2}g^2V_1^2(U_\mu^+U_\mu^- + U_\mu^-U_\mu^+) + \tfrac{1}{2}g^2V_3^2(V_\mu^+V_\mu^- + V_\mu^-V_\mu^+)\end{aligned} \tag{32}$$

From equation (32) we notice that C_μ, D_μ , E_μ, $U_\mu^\pm$, and $V_\mu^\pm$ have acquired masses and thus the symmetry is broken down to $SU(2) \times U(1)$. We also notice that the bosons C_μ and E_μ have equal masses and the D_μ boson is the heaviest of all these particles. Its mass is the sum of the masses of C_μ and E_μ bosons.

4. *CP* VIOLATION IN *K* DECAYS

To discuss the *CP* violation in the present theory, following Deshpande *et al.* (1977), we identify the states K_1^0 and K_2^0 as follows:

$$K_1^0 = \bar{d}\gamma^\mu s + \bar{s}\gamma^\mu d \tag{33}$$

$$K_2^0 = i(\bar{s}\gamma^\mu d - \bar{d}\gamma^\mu s) \tag{34}$$

In equation (26) these states then mix through E'_μ and D'_μ to form K_S^0 and K_L^0.

Looking at equation (24), we see that K_1 is coupled to E_μ and K_1 and K_2 to D_μ only, since there is no mixing of K_1 and K_2 hence no formation of the states K_s and K_L, consequently no *CP* violation exists. However, our definitions D'_μ and E'_μ by (25) yield (26), in which we find that the linear combinations of the states K_1 and K_2 are coupled to D'_μ and E'_μ. The implications of our definition will be discussed in Section 5.

In equation (26) we have the terms

$$-\tfrac{1}{2}gE'_\mu[\cos(\theta/2)\,(\bar{d}_L\gamma^\mu s_L+\bar{s}_L\gamma^\mu d_L) + e^{i\phi}\sin(\theta/2)\,i(\bar{s}_L\gamma^\mu d_L-\bar{d}_L\gamma^\mu s_L)] \tag{35}$$

$$-\tfrac{1}{2}gD'_\mu[-e^{-i\phi}\sin(\theta/2)\,(\bar{d}_L\gamma^\mu s_L+\bar{s}_L\gamma^\mu d_L) + \cos(\theta/2)\,i(\bar{s}_L\gamma^\mu d_L-\bar{d}_L\gamma^\mu s_L)] \tag{36}$$

In the above expressions we reparametrize θ and ϕ in terms of ε_1 and ε_2 as follows:

$$\varepsilon_1=e^{i\phi}\tan(\theta/2), \qquad \varepsilon_2=-e^{-i\phi}\tan(\theta/2) \tag{37}$$

where ε_1 and ε_2 are complex numbers.

In terms of ε_1 and ε_2, expressions (35) and (36) can be recast as follows:

$$-\tfrac{1}{2}gE'_\mu\frac{1}{(1+|\varepsilon_1|^2)^{1/2}}(|K_1\rangle+\varepsilon_1|K_2\rangle) \tag{38}$$

$$-\tfrac{1}{2}gD'_\mu\frac{1}{(1+|\varepsilon_2|^2)^{1/2}}(|K_2\rangle+\varepsilon_2|K_1\rangle) \tag{39}$$

where we define K_S^0 and K_L^0 as

$$|K_S^0\rangle=\frac{1}{(1+|\varepsilon_1|^2)^{1/2}}(|K_1\rangle+\varepsilon_1|K_2\rangle) \tag{40}$$

$$|K_L^0\rangle=\frac{1}{(1+|\varepsilon_2|^2)^{1/2}}(|K_2\rangle+\varepsilon_2|K_1\rangle) \tag{41}$$

Furthermore, if $\varepsilon_1=\varepsilon_2=\varepsilon$, $|K_s^0\rangle$ and $|K_L^0\rangle$ can be rewritten in terms of $|K^0\rangle$ and $|\bar{K}^0\rangle$ as

$$|K_s^0\rangle=\frac{1}{[2(1+|\varepsilon|^2)]^{1/2}}[(1+\varepsilon)|K^0\rangle-(1-\varepsilon)|\bar{K}^0\rangle] \tag{42}$$

$$|K_L^0\rangle=\frac{1}{[2(1+|\varepsilon|^2)]^{1/2}}[(1+\varepsilon)|K^0\rangle+(1-\varepsilon)|\bar{K}^0\rangle] \tag{43}$$

which are the expressions indicating *CP* violation and *CPT* invariance (see, for example, Commins, 1973).

Granting that D'_μ and E'_μ are the appropriate gauge states exchanged in the direct transition from K_1 to K_2, one can calculate the magnitude of the effective *CP* violation, which will be reported elsewhere. The point we want to make here is the relevance of the Cabibbo angle to *CP* violation. We will first establish that given the expressions coupled to D'_μ and E'_μ in equation (26), which are the linear combinations of K_1 and K_2 states, it is not possible to decouple them by a redefinition of quark phases, and later we show that if the Cabibbo angle does not exist, the combinations decouple; K_1 couples to E_μ and K_2 to D_μ only.

Let us define a new state $\hat{s} = e^{-i\delta}s$, which is just a phase change and does not introduce any new physics in the theory. Now K_1 and K_2 can be defined in terms of d and $\hat{s}$ quarks as follows:

$$K_1 = \bar{d}\gamma^\mu \hat{s} + \bar{\hat{s}}\gamma^\mu d \tag{44}$$

$$K_2 = i(\bar{\hat{s}}\gamma^\mu d - d\gamma^\mu \hat{s}) \tag{45}$$

Then the expressions containing D'_μ and E'_μ in (15) can be recast as

$$-\tfrac{1}{2}gE'_\mu(\bar{\hat{s}}_L\gamma^\mu d_L + \bar{d}_L\gamma^\mu \hat{s}_L) \tag{46}$$

$$-\tfrac{1}{2}gD'_\mu i(\bar{\hat{s}}_L\gamma^\mu d_L - \bar{d}_L\gamma^\mu \hat{s}_L) \tag{47}$$

which means that now D'_μ couples to K_2 and E'_μ couples to K_1 only; hence there will be no formation of K_S and K_L states and consequently the theory remains CP-invariant. However, such a procedure introduces a complex phase in the coupling terms of $U^\pm_\mu$, $V^\pm_\mu$, and $W^\pm_\mu$; for instance, we will have $U^+_\mu(\bar{u}_L\gamma^\mu \hat{s}_{cL}\, e^{+i\delta} - \overline{c_L}\gamma^\mu d_{cL})$, which cannot be absorbed in the quark field u. This can be seen as follows.

In the Lagrangian (24), if $\hat{u}$ is defined as $u = \hat{u}\, e^{-i\delta}$, the expressions coupled to U^+_μ and $W^\pm_\mu$ have to be recast as

$$\begin{aligned} &\frac{1}{\sqrt{2}} gU^+_\mu(\bar{\hat{u}}_L\gamma^\mu \hat{s}_{cL} - \bar{c}_L\gamma^\mu \hat{s}\, e^{i\delta}) \\ &\frac{1}{\sqrt{2}} gW^+_\mu(\bar{\hat{u}}_L\, e^{i\delta}\gamma^\mu d_{cL} + \bar{c}_L\gamma^\mu \bar{\hat{s}}_{cL}\, e^{i\delta}) \end{aligned} \tag{48}$$

For the Lagrangian to remain invariant in terms of new states, the phases have to be absorbed in c and d by defining $d = \hat{d}\, e^{+i\delta}$ and $c = \hat{c}\, e^{-i\delta}$. Thus, with the introduction of the new states defined above, i.e.,

$$\hat{u} = u\, e^{i\delta}, \qquad \hat{d} = d\, e^{-i\delta}, \qquad \hat{s} = s\, e^{-i\delta}, \qquad \hat{c} = c\, e^{i\delta} \tag{49}$$

and by their insertion in the expressions coupled to D'_μ and E'_μ in equation (26), we recover the original expressions in terms of the new states. Thus, the coupling of K_1 and K_2 cannot be removed by a redefinition of quark states with the additional phases.

We now show that if the Cabibbo angle does not exist, there will be no CP violation in K decays. Putting θ_c equal to zero in equation (23), we find $\breve{E}_\mu = E_\mu$ and $\breve{C}_\mu = C_\mu$. Equation (25) defines D'_μ and E'_μ in terms of

$\breve{E}_\mu(=E_\mu)$ and D_μ. Substituting D'_μ and E'_μ in Equation (26) in terms of E_μ and D_μ, we find that the expressions containing D'_μ and E'_μ reduce to

$$-\tfrac{1}{2}gD_\mu|K_2\rangle \tag{50}$$

$$-\tfrac{1}{2}gE_\mu|K_1\rangle \tag{51}$$

From equations (50) and (51) we see that K_1 and K_2 have decoupled and thus the states K_S^0 and K_L^0 no longer exist. Hence, there will be no *CP* violation. We thus see that the existence of the Cabibbo angle emerges as a necessary condition for *CP* violation in *K* decays.

5. DISCUSSION OF RESULTS

From equations (33), (34), and (24) we observe that K_1 couples to $\breve{E}_\mu$ and K_2 couples to D_μ. However, in equation (23), $\breve{E}_\mu$ is defined in terms of C_μ and E_μ; in fact, this definition is a necessary consequence of K_1 being coupled to both C_μ and E_μ, which we have already established to be equally massive; on the other hand, however, in defining D'_μ and E'_μ there is no built-in mechanism in the theory to necessitate such a mixing of $\breve{E}_\mu$ and D_μ of differing masses; it is only compatible with the $SU(2)$ invariance of the theory. If $\theta=0$ and $\phi=0$, from (25) D'_μ reduces to D_μ and E'_μ to $\breve{E}_\mu$, and from equation (26) we see that K_1 couples to $\breve{E}_\mu$ and K_2 to D_μ only; hence, no mixing of K_1 and K_2 occurs. The parameters θ and ϕ determine the magnitude of the mixing of the states K_1 and K_2, and from (37), are related to the parameters ε_1 and ε_2, which are equal if the theory is to be *CPT*-invariant. From equations (42) and (43) we see that ε is the *CP*-violating and *CPT*-conserving parameter. The contributions of *CP* violation in the direct transitions from K_1 to K_2 mediated by the exchange of D'_μ and E'_μ do not cancel, as they are nondegenerate; hence, their mass difference may be regarded as the origin of *CP* violation in this theory.

If θ_c were zero, as we have shown, no formation of D'_μ and E'_μ would take place to mix, and hence there will be no *CP* violation in the theory. Thus, the existence of the Cabibbo angle emerges as a necessary condition in *K* decays.

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